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TEACHING MATERIAL ON

MATHEMATICS

SCHOOL OF SCIENCE

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In graphical method, we examine the extreme points of the feasible region/space for optimal solution at one of them when two variables or three variables are involved. For LP problems with several variables, we may not be able to graph the feasible region, but surely the optimal solution will still lie anywhere at an extreme point of the many sided, multidimensional figure (multi n-dimension polyhedron) that represents the feasible solution space. The Simplex method also called iterative method examines iteratively the extreme points in a systematic manner (as an Algorithm) until an optimal solution is reached.

By simplex as a term we mean an object in n-dimensional space connecting n+1 points such as a line segment in one dimensional space connecting two points, it is a triangular region/space in two dimensions and a four sided pyramidal shaped region/space in a three dimensional space and so on.

A more efficient method to suggest an optimal soln for such LP problems (for more than two variables) was first developed by G. B. Dantzig in 1947 as an algorithm which consists of the following steps:—

(i) Finding a trial B.F. Solution of the L.P.P.
(ii) Testing whether it is an optimal soln or not.
(iii) Improving the first trial B.F.S (if it is not optimal) by a set of rules.
(iv) Repeating the step (ii) & (iii) we get an optimal soln / bounded soln.

Computational Procedure (94) of the simplex method for the solution of a Maximization L.P.P.

Step-1: If the problem is of minimization, convert it into the maximization problem.

The value of $x_1, x_2, \ldots x_n$ that minimize the function problem $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$, maximize the function:—

$$Z^* = -Z = -c_1x_1 - c_2x_2 - \cdots - c_nx_n.$$

Step-2.. Make all the b_i's positive :- If any of the b_i's is negative, multiply the corresponding constraints by -1. By this multiplication the inequality is also reversed.

Step-3:- (a) Convert the constraints into equations by introducing the non-negative slack or surplus variables.

Also introduce artificial variables to the constraints where surplus variables are inserted and which do not form the columns of unit matrix.

Step-4:- To find initial basic feasible solution (BFS):—

~~Case~~ In the constraints of a general L.P.P there may be any of three sign $\le = \ge$ as follows:—

Find $x_1, x_2, x_3 \ldots x_n$ which optimize the Linear fn

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n \quad \text{———} (1)$$

S.T.C. $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\le = \ge)\ b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\le = \ge)\ b_2$ — (2)

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\le = \ge)\ b_m$

and non-negativity constraint $x_j \ge 0,\ j = 1 \ldots n$ — (3)

Some Denotations:—

where all a_{ij}'s, b_i's and c_j's are constants and $(x_1, x_2 \cdots x_n)$ are variables.

Which in matrix form we write :- $\bar{z} = \bar{c}\bar{x}$

S.T.C $A\bar{x} \ (\leq = \geq) \ \bar{b}$

$\bar{x} \geq 0$

where $\bar{A} = [a_{ij}]_{m \times n}$ is the matrix of coeff. of order $m \times n$.

$\bar{c} = (c_1 c_2, c_3 \cdots c_n)_{1 \times n}$ is a row vector called "price vector",

$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = [x_1, x_2, \cdots x_n]'$ is column vector of variable

$\bar{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1} = [b_1, b_2, \cdots b_m]'$ is column vector called "requirement" vector

$\alpha_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$ is the column vector formed by the coefficients of x_j in all the constraints and is denoted by α_j.

Then $\bar{A} = (\alpha_1, \alpha_2, \cdots \alpha_n)$

We denote by $\bar{B}$ a $m \times m$ non-singular matrix whose column vectors are linearly independent columns of the matrix A. If these columns are denoted by $\beta_1, \beta_2, \cdots \beta_m$ then

$\bar{B} = (\beta_1, \beta_2, \cdots, \beta_m)$

Matrix B is called the basis matrix.

Thus the variables corresponding to $\beta_1, \beta_2, \cdots \beta_m$ are called the "basic variables" and will be denoted as $x_{B_1}, x_{B_2}, x_{B_3} \cdots x_{B_m}$ respectively.

The vector (column vector) of these m basic variables, is denoted by x_B or X_B ie, $X_B = [x_{B_1}, x_{B_2}, \cdots, x_{B_m}]$

where $\boxed{\bar{X}_B = B^{-1}\bar{b}}$ Called the B.F.S of the LPP (Basic feasible soln of the LPP)

we denote $z_j = C_B Y_j = (c_{B_1} y_{1j} + c_{B_2} y_{2j} + \cdots + c_{B_m} y_{mj})$

$\Delta_3 = c_3 - c_B Y_3 = 4 - (0, 0, 0)(0, 5, 4) = 4$

To Determine Starting Initial B.F.S : (Contd) 96

Assuming that the requirement vector ≥ 0 (b) we have three cases to find starting B.F.S :

Case-1 When all the original constraints have $\leq$ sign:

Convert all into eqns by inserting slack variables only

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + 1\cdot x_{n+1} + 0 + \cdots + 0 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + 0\cdot x_{n+1} + 1\cdot x_{n+2} + 0 + \cdots + 0 = b_2$$

$$\cdots\cdots\cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + 0 + 0 + \cdots + 1\cdot x_{n+m} = b_m$$

Here $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$ are slack variables.

In matrix form these equations can be written as.

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & \cdots & 1 \end{array}\right]_{\text{slack vectors}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Here we take the initial basis matrix $B = I_m$ (unit matrix of order m)

Hence the initial basic solution is given by

$$X_B = B^{-1}\bar{b} = I_m\bar{b} = \bar{b} \geq 0$$

Thus the initial B.F.S is

$$x_{n+1} = x_{B1} = b_1,\; x_{n+2} = x_{B2} = b_2, \ldots, x_{n+m} = x_{Bm} = b_m$$

which can be obtained by writing all the non-basic variables (i.e., given variables) $x_1, x_2 \cdots x_n$ all equal to zero and solving the equations for the remaining variables (i.e., slack variables) $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$

Case-II To find B.F.S when constraints (original) have sign '≥' :–

Convert all the constraints into equations by inserting surplus variables as under :–

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n - x_{n+1} = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n - x_{n+2} = b_2$$
$$\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n - x_{n+m} = b_m$$

Here $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$ are surplus variables.

Writing in matrix form we have

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & -1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & -1 & \cdots & 0 \\ \cdots & & & & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & \underbrace{0 \quad 0}_{\text{Surplus Vector}} & & \cdots & -1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

Here taking the initial basis matrix $B = -I_m$, we have $\bar{x}_B = B^{-1}\bar{b} = -I_m\bar{b} = -\bar{b} \not\geq 0$, Hence this basic soln is not B.F.S. In order to avoid this difficulty we add one more variable to each constraints. These variables are called "Artificial variables".

Adding surplus & artificial variables, the constraints of a given L.P.P are transformed to the following equations:–

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n - x_{n+1} + \cdots + x_{n+m+1} = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \cdots - x_{n+2} + x_{n+m+2} = b_2$$
$$\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \cdots - x_{n+m} + x_{n+m+m} = b_m$$

Here $x_{n+m+1}, x_{n+m+2}, \ldots, x_{n+m+m}$ are the artificial variables. In the matrix form these eqns can be written as :–

$\Delta_3^2 = c_3 - c_B Y_3 = 4 - (0,0,0)(0,5,4) = 4$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \hline a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & \cdots & & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \hline 0 & 0 & \cdots & 1 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

Here we can take the basis matrix $B = I_m$

$\therefore \vec{X}_B = B^{-1}\vec{b} = I_m^{-1}\vec{b} = \vec{b} \geq 0$ which is a B.F.S.

and $x_{n+m+1} = x_{B_1} = b_1,\ x_{n+m+2} = x_{B_2} = b_2, \cdots, x_{n+m+m} = x_{Bm} = b_m$

which can be obtained by writing all the non-basic variables (i.e, given variables) $x_1, x_2, \cdots x_n, x_{n+1}, x_{n+2}, \cdots, x_{n+m}$ equal to zero and solving the equations for remaining basic variables (i.e, artificial variables) $x_{n+m+1}, \cdots, x_{n+m+m}$.

Case-III When constraints have ~~equal~~ '$\leq$', '$\geq$' and '$=$' sign

In this case the constraints are converted into equations by inserting slack, surplus and artificial variables. Here the basis matrix $B = I_m$ is obtained by taking the slack and artificial ~~variables~~ vectors.

In this case also the initial B.F.S is obtained by writing all the non-basic variables (i.e, the given variables and surplus variables) equal to zero and solving for remaining basic variables (artificial variables).

$\therefore X_B = b \geq 0.$

• Case-II & III are same.

Step-4 (contd.): ~~After finding the~~ (99) (4) 99

If in a LPP artificial variables are also introduced in step-3 then we follows two-phase method. ~~[illegible]~~ In phase-I we proceed to get the starting B.F.S in terms of non-artificial variables. Big-M-method can also be used in such cases.

Step-5. Construction of starting simplex Table as follows.

Starting Simplex Table.

B	C_B	c_j / X_B	c_1 coeff. of x_1 / $Y_1(=\alpha_1)$	c_2 coeff. of x_2 / $Y_2(=\alpha_2)$	...	c_k / $Y_k=(\alpha_k)$	...	c_n / $Y_n=(\alpha_n)$	c_{n+1} / $Y_{n+1}(\beta_1)$	c_{n+2} / $Y_{n+2}(\beta_2)$	...	c_{n+m} / $Y_{n+m}(\beta_m)$	Mini Ratio X_B/Y_k
$\beta_1=Y_{n+1}$	$C_{B_1}=c_1$	$X_{B_1}=b_1$	$y_{11}=a_{11}$	$y_{12}=a_{12}$	...	$y_{1k}=a_{1k}$	...	$y_{1n}=a_{1n}$	1	0	—	0	
$\beta_2=Y_{n+2}$	$C_{B_2}=c_2$	$X_{B_2}=b_2$	$y_{21}=a_{21}$	$y_{22}=a_{22}$		$y_{2k}=a_{2k}$		$y_{2n}=a_{2n}$	0	1		0	
⋮	⋮	⋮							⋮				
$\beta_r=Y_{n+r}$	$C_{B_r}=c_r$	$X_{B_r}=b_r$	$y_{r1}=a_{r1}$	$y_{r2}=a_{r2}$		$\boxed{y_{rk}=a_{rk}}$		$y_{rn}=a_{rn}$	⋮				
$\beta_m=Y_{n+m}$	$C_{B_m}=c_m$	$X_{B_m}=b_m$	$y_{m1}=a_{m1}$	$y_{m2}=a_{m2}$		$y_{mk}=a_{mk}$		$y_{mn}=a_{mn}$	0	0		1	
$Z=C_BX_B=0$	Δ_j		Δ_1	Δ_2		Δ_k	—	Δ_n	Δ_{n+1}	Δ_{n+2}	...	Δ_{n+m}	

Here the column corresponding to the coefficients of $x_1, x_2, \dots x_n$ are shown by $Y_1, Y_2 \dots$

Step-6. Test of starting B.F.S for optimality :— This is done by computing an evaluation of Δ_j for each variable x_j by the formula $\Delta_j = c_j - C_B Y_j$. The value of $\Delta_j = 0$, if x_j is a basic variable.

Insert the values of Δ_j's for all variables as a row in the starting simplex table.

Here c_j is the row of the coefficients of the variables in the objective function.

$\Delta_3 = c_3 - C_B Y_3 = 4 - (0,0,0)(0,5,4) = 4$

(i) If $\Delta_j \le 0$ for each j (i.e., no $\Delta_j > 0$) the solution under test is optimal.

(a) If none of Δ_j is positive, but any are zero (for non-basic variables), then other optimal solution exist with the same value of Z.

(b) If all of Δ_j are negative (for all non-basic variables), the solution under test is unique optimal solution.

(ii) If $\Delta_j > 0$ for any j i.e., if one or more Δ_j are positive the solution under test is not optimal. Then we must proceed to the next step-7

(c) If corresponding to maximum positive Δ_j, all the elements in the column y_j are negative or zero, then the solution under test will be unbounded.

(d) If the value of at least one artificial variable appearing in the basis is non-zero and the optimality condition is satisfied, then we shall say that the problem has no feasible solution.

Step-7. To find incoming (or entering) and outgoing vectors: — To improve the above solution (which is not optimal) we find the vector entering the basis matrix (called in-coming vector) and the vector to be removed from the basis matrix (called outgoing vector) by the following rules.

To find incoming vector: — The incoming vector will be taken as α_k if $\Delta_k = \text{Max}\, \Delta_j$.

To find in-coming vector (contd....) :- (5) 58

(10)

If max value of Δ_j occurs at more than one α_j, then any one of these may be taken as an incoming vector.

To find out going vector :- The out-going vector β_r is taken corresponding to the value of r for which
$$\frac{x_{Br}}{y_{rk}} = \text{Min}_i \left\{ \frac{x_{Bi}}{y_{ik}} , y_{ik} > 0 \right\}.$$
where α_k is the in-coming vector.

If minimum is not unique, the minimum occurs for more than one value of i, then more than one variable will vanish in the next solution so the next soln will become a degenerate B.F.S., for which outgoing vector is selected in a different way.

Step-8 :- When $\alpha_k (= Y_k)$ is the incoming vector and $Y_r (= \beta_r)$ the outgoing vector then the element $Y_{rk} (= a_{rk})$ is called the key element or pivot element which is at the intersection of minimum ~~ratio arrow~~ ratio arrow ($\rightarrow$) and incoming vector arrow ($\uparrow$). We mark this element in $\square$.

In order to bring $a_k (= Y_k)$ in place of $Y_r (= \beta_r)$ there should be unity at the position $\square$ i.e. the key element $Y_{rk} (= a_{rk})$ should be equal to (1) unity. If it is not 1 then divide all the elements of this row by this key element a_{rk}. Then subtract appropriate multipliers of this row (containing key element) from all the other rows and obtain zero (= 0) at all other position of this column.

$\Delta_3 = c_3 - c_B Y_3 = 4 - (0,0,0)(0,3,4) = 4, \ldots$

column $\alpha_K (= Y_K)$. Now bring $Y_K (\alpha_K)$ in place of Y_r (α_r i.e., β_r) and construct new (revised) simplex table.

In this way we get improved basic feasible solution.

Step-9. Now test the above B.F.S for optimality as in step-6.

If this soln is not optimal then repeat step (7) and (8) in succession, until an optimal solution is finally obtained.

Example-1. Solve the L.P. Problem.

Maximize $Z = 3x_1 + 5x_2 + 4x_3$. Subject to the constt.

$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15 \quad \text{and } x_1, x_2, x_3 \ge 0.$$

Soln.

Step-1 The problem is a problem of maximization.

Step-2 All the b_i's are already +ve.

Step-3 Now the inequalities are converted to eqns. by the introduction of slack variables x_4, x_5, x_6 as follows.

$$2x_1 + 3x_2 + 0\cdot x_3 + x_4 = 8$$
$$0\cdot x_1 + 2x_2 + 5x_3 + \quad + x_5 = 10$$
$$3x_1 + 2x_2 + 4x_3 + \quad + x_6 = 15$$

Step-4 Taking $x_1 = 0, x_2 = 0, x_3 = 0$ we get $x_4 = 8, x_5 = 10, x_6 = 15$ which is the starting B.F.S.

Step-5. Now we construct the starting simplex table:-

Step-6:- Here we compute Δ_j for all zero variables (non-basic) $x_j, j = 1,2,3$ by the formula $\Delta_j = c_j - C_B y_j$.

Table-1

	c_j		$c_1=3$	$c_2=5$	$c_3=4$	$c_4=0$	$c_5=0$	$c_6=0$	Min Ratio
y_B	C_B	X_B	Y_1 (α_1)	Y_2 (α_2)	Y_3 (α_3)	$Y_4 = x_4$ coeff. (β_1)	$Y_5 = x_5$ coeff. (β_2)	$Y_6 = x_6$ coeff. (β_3)	$\frac{X_B}{Y_2}$
y_4	0	8	2	$\boxed{3}$	0	1	0	0	8/3 Min →
y_5	0	10	0	2	5	0	1	0	10/2 = 5
y_6	0	15	3	2	4	0	0	1	15/2
$Z = C_B X_B = 0$		Δ_j	$\Delta_1 = 3$	$\Delta_2 = 5$ ↑	4	0 ↓	0	0	

Now, $\Delta_3 = c_3 - C_B Y_3 = 4 - (0,0,0)(0,5,4) = 4, 0,0$

To find outgoing vector: Since $Y_2(a_2)$ is incoming vector therefore we consider the

$$\frac{X_B}{Y_2} = \left(\frac{X_{B1}}{y_{12}}, \frac{X_{B2}}{y_{22}}, \frac{X_{B3}}{y_{32}}, y_{i2} > 0\right) = \left(\frac{8}{3}, \frac{10}{2}, \frac{15}{2}\right)$$

$$\text{Since } \frac{X_{Br}}{Y_{r2}} = \underset{i}{\text{Min}}\left[\frac{X_{Bi}}{Y_{i2}}, y_{i2} > 0\right] = \text{Min}\left(\frac{X_{B1}}{y_{12}}, \frac{X_{B2}}{y_{22}}, \frac{X_{B3}}{y_{32}}\right)$$

$$= \frac{8}{3} = \frac{X_{B1}}{y_{12}}$$

$r = 1$ i.e. $\beta_1 (= Y_4)$ is the outgoing vector.

Step-8: Since $Y_2(\alpha)$ is incoming vector and $\beta_1(Y_4)$ outgoing vector,

∴ the key element is $y_{12} (= a_{12})$ as shown in the table which is equal to 3. (not equal to 1)

∴ In order to bring Y_2 in place of Y_4 in the basis we make the following operations:

Divide the 1st row containing key element $a_{12} = 3$ by 3, to get unity at this position and then subtract 2 times of the first row (obtained after dividing by 3) from the second and the third rows and subtract 5 times of this row from the row of Δ_j's to get the new value of Δ_j's.

Thus we have the second simplex table in which $\beta_1 (= Y_4)$ is replaced by $\alpha_2 (= Y_2)$ as follows: —

Table-2 (105) (7) 52

	c_j	3	5	4	0	0	0	Min Ratio
C_B	X_B	Y_1 (α_1)	Y_2 (β_1)	Y_3 (α_3)	Y_4 (α_4)	Y_5 (β_5)	Y_6 (β_6)	X_B/Y_3
Y_2 5	8/3	2/3	1	0	1/3	0	0	— x
Y_5 0	14/3	-4/3	0	$\boxed{5}$	-2/3	1	0	14/15 → (Min)
Y_6 0	29/3	5/3	0	4	-2/3	0	1	29/12
$z = C_B X_B = \frac{40}{3}$	Δ_j	-1/3	0	4	-5/3	0	0	

↑ incoming vector ↓ outgoing vector

To check the values of Δ_j, we also compute Δ_j's by using the formula $\Delta_j = c_j - C_B Y_j$ for $x_1, x_3 \& x_4$

$$\Delta_1 = c_1 - C_B Y_1 = 3 - (5,0,0)\cdot\left(\tfrac{2}{3}, -\tfrac{4}{3}, \tfrac{5}{3}\right) = 3 - \tfrac{10}{3} = -\tfrac{1}{3}$$

$$\Delta_3 = c_3 - C_B Y_3 = 4 - (5,0,0)\cdot(0,5,4) = 4$$

$$\Delta_4 = c_4 - C_B Y_4 = 0 - (5,0,0)\left(\tfrac{1}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}\right) = -\tfrac{5}{3}$$

The values of $\Delta_2, \Delta_5, \Delta_6$ will be zero as they corresponds to unit column vectors.

Since in table Table-3 all Δ_j are not less than or equal to zero, therefore this sol^n is also not optimal.

Since $\Delta_3 = 4$ is max of the Δ_j's

$\alpha_3 (= Y_3)$ is the incoming vector

Also $\frac{x_{Br}}{y_{r3}} = \text{Min}\left[\frac{x_{Bi}}{y_{i3}}, y_{i3} > 0\right]$

$= \text{Min}\left[\frac{x_{B2}}{y_{23}}, \frac{x_{B3}}{y_{33}}\right]$, $\frac{x_{B1}}{y_{13}}$ not considered $\because y_{13} = 0$

$= \text{Min}\left[\frac{14}{15}, \frac{29}{12}\right] = \frac{14}{15} = \frac{x_{B2}}{y_{23}} \Rightarrow r = 2$

$\Delta_3 = c_3 - C_B Y_3 = 4 - (0,0,0)\cdot(0,5,4) = \ldots$

i.e, $\beta_2(=Y_5)$ is the outgoing vector and $y_{23}=a_{23}=5$ is the key element. In order to bring Y_3 in place of $\beta_2(=Y_5)$, we divide the 2nd row containing the key element by 5 to get 1 at this position, then subtract 4 times of the second row thus obtained from third row and also from the row of Δ_j's.

The third simplex table in which $\beta_2(=Y_5)$ is replaced by Y_3 as follows.

Table-3

		C_j	3	5	4	0	0	0	
B	C_B	X_B	Y_1 (α_1)	Y_2 (α_2)	Y_3 (β_2)	Y_4 (α_4)	Y_5 (α_5)	Y_6 (β_3)	X_B/Y_1
Y_2	5	8/3	2/3	1	0	1/3	0	0	4
Y_3	4	14/15	-4/15	0	1	-2/15	1/5	0	-7/2 (Neg)
Y_6	0	89/15	41/15	0	0	-2/15	-4/5	1	89/41 (Mini)
$Z=C_BX_B = 256/15$		Δ_j	11/15	0	0	-17/15	-4/5	0	

incoming vector Outgoing vector

To check the values of Δ_j we again compute Δ_j by using the formula $\Delta_j = C_j - C_B Y_j$.

$$\Delta_1 = C_1 - C_B Y_1 = 3 - (5,4,0)\cdot(2/3, -4/15, 41/15)$$
$$= 3 - (10/3 - 16/15) = 11/15$$

$$\Delta_4 = C_4 - C_B Y_4 = 0 - (5,4,0)\cdot(1/3, -2/15, -2/15) = -17/15$$

$$\Delta_5 = C_5 - C_B Y_5 = 0 - (5,4,0)\cdot(0, 1/5, -4/5) = -4/5$$

Also $\Delta_2 = 0 = \Delta_3 = \Delta_6$.

Since all Δ_j are not less than or equal to zero

$\therefore$ the soln is not optimal.

Since all Δ_j are not less than or equal to zero. 5
∴ we have to repeat the process again.
Since Δ_1 is max of all the Δ_j's
∴ $\alpha_1 (= Y_1)$ is the incoming vector

Also
$$\frac{x_{Br}}{y_{r1}} = \underset{i}{\text{Min}} \left[\frac{x_{Bi}}{y_{i1}}, y_{i1} > 0 \right]$$
$$= \text{Min} \left[\frac{x_{B1}}{y_{11}}, \frac{x_{B3}}{y_{31}} \right], \frac{x_{B2}}{y_{21}} \text{(not considered)}$$
$$= \text{Min} \left[4, \frac{89}{41} \right] = \frac{89}{41} = \frac{x_{B3}}{y_{31}} \quad \boxed{\therefore r = 3}$$

i.e., $\beta_3 (= Y_6)$ is the outgoing vector and $y_{31} = a_{31} = 14/15$ is the key element. ~~the key element is~~

Again in order to bring the Y_1 in place of $\beta_3 (= Y_6)$ we divide the 3rd row containing the key element by 41/15 to get 1 at this position then subtract 2/3 times of the third row (thus obtained) from 1st row, and add 4/15 times of third row to the 2nd row and subtract 11/15 times of third row from the row of Δ_j's. Then the fourth simplex table in which $\beta_3 (= Y_6)$ is replaced by Y_1 is as follow -

Table - 4

		c_j	3	5	4	0	0	0	Min Ratio.
B	C_B	X_B	$Y_1(\beta_3)$	$Y_2(\beta_1)$	$Y_3(\beta_2)$	$Y_4(\alpha_4)$	$Y_5(\alpha_5)$	$Y_6(\alpha_6)$	
Y_2	5	$\frac{50}{41} = x_1$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$-\frac{10}{41}$	
Y_3	4	$\frac{62}{41} = x_2$	0	0	1	$-6/41$	$\frac{5}{41}$	$4/41$	
Y_1	3	$89/41 = x_3$	1	0	0	$-2/41$	$-12/41$	$15/41$	
$z = C_B X_B = \frac{756}{41}$		Δ_j	0	0	0	$-45/41$	$-24/41$	$-11/41$	

Δ_j where. $\Delta_3 = c_3 - C_B Y_3 = 4 - (0,0,0)(0,5,4) = h, 0,0$

To check the value of Δ_j's we also compute Δ_j by using the formula

$\Delta_j = c_j - C_B Y_j$.

$\Delta_4 = c_4 - C_B Y_4 = 0 - (5,4,3)\left(\frac{15}{41}, \frac{-6}{41}, \frac{-2}{41}\right) = -\frac{45}{41}$

Similarly $\Delta_5 = -24/41$, $\Delta_6 = -11/41$, $\Delta_1 = 0 = \Delta_2 = \Delta_3$

Since all the Δ_j's for zero variables (non-basic variables) are negative so this soln is optimal.

Optimal soln is $x_1 = 89/41$, $x_2 = 50/41$, $x_3 = 62/41$

And Max $Z = $ Rs $765/41$

All table can be computed in a single table as Table-5

Table 5 (Left for students)

Examples Solve by simplex method the following L.P. problem.

Minimize $Z = x_1 - 3x_2 + 2x_3$

Sub. to $3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$.

Flow chart of simplex ~~Method~~ Algorithm

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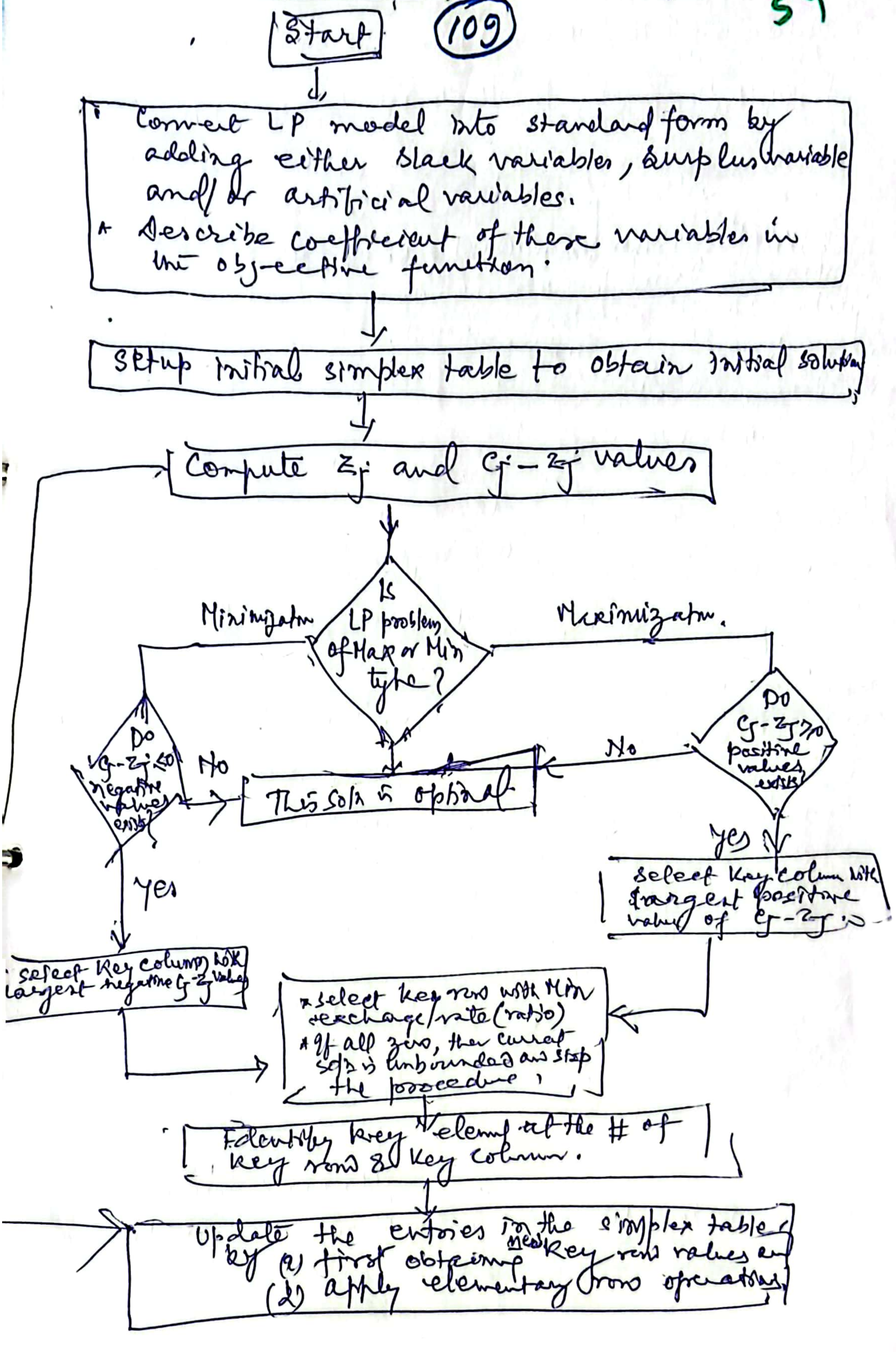

iv) artificial ...

$\Delta_3 = C_3 - C_B Y_3 = 4 - (0,0,0)(0,5,4) = \ldots$